Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 3**

1

*X,x*

2

3

*Z,z*





2

1

Electric current causes heat generation in the bar shown. Calculate the temperature at the centre if the wall temperature (nodes 1 and 2) is . Cross sectional area , thermal conductivity , and heat production rate per unit length  are constants.

**Solution template**

In a pure heat conduction problem, density expressions of the bar model are given by

 and 

in which  is the temperature,  the thermal conductivity, and  the rate of heat production (per unit length).

For bar 1, the nodal temperatures are  and  of which the latter is unknown. With a linear interpolation to temperature (notice that variation of  vanishes)

  ,

  .

When the approximation is substituted there, density expression  simplifies to

,

Virtual work expression is the integral of the density over the element domain

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The nodal temperatures of bar 2 are  and . Linear interpolation gives (variations of the given quantities like  vanish)

  ,

  .

When the approximation is substituted there, density expression  simplifies to

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Element contribution to the variational expressions is the integral of density over the element domain

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Variational expression is sum of the element contributions

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Variation principle   and the fundamental lemma of variation calculus give

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